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WATERTOWN ARSENAL LABORATORIES

INITIAL DRIVING EDGE PRESSURES OF A ROTATING BAND

TECHNICAL REPORT NO. WAL TR 760.3/1

BY

JOHN CAMPO

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IMPROVEMENT OF ARTILLERY AMMUNITION - LONG RANGE PROGRAM

D/A PROJECT 504-03-061

WATERTOWN ARSENAL
WATERTOWN 72, MASS.

Rotating bands -
pressures

Rifling

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TITLE

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ABSTRACT

Driving edge pressures during initial stages of engraving have not been considered in the past in the design of rotating bands. Recent tests indicate that these pressures may be sufficiently high to produce cracking of chromium plating and subsequent accelerated gun tube erosion in the region of the origin of rifling.

In this report the general differential equations of motion defining the force equilibrium in the axial and rotational directions as a function of time have been derived. Simultaneous solution of these equations leads to a general expression, applicable to general twist rifling, for determining initial driving edge pressures. Further modifications of this expression are made specifically for uniform twist rifling. Finally, additional interrelations of terms appearing in the equations dealing with uniform twist rifling are supplied so that these individual terms as well as those of driving edge pressures may be computed.

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APPROVED:

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Director
Watertown Arsenal Laboratories

CONTENTS

NOMENCLATURE

(See also Figures 1-4)

A	=	$\dfrac{K_2 \, r_m \, C_1^2}{I}$	
A_b	=	Area at base of projectile	(in^2)
A_{de}	=	Driving edge area of rotating band (See Figure 1)	(in^2)
$(A_{de})_{proj}$	=	Projected driving edge area of rotating band (See Figure 1)	(in^2)
B	=	$b \, C_1 \, m \, (m - 1)$	
C_1	=	Coefficient dependent on relationship between x and t, i.e., $x = C_1 \, t^m$	
C_2	=	Coefficient dependent on relationship between P_g and t, i.e., $P_g = C_2 \, t^w$	
D_b	=	Bore diameter of gun	(in)
D_g	=	Diameter of gun groove	(in)
D_ℓ	=	Diameter of gun land (Note: D_b and D_ℓ are identical)	(in)
F_b	=	Force at outside surface of band due to band pressure (See Figure 4)	(lb)
F_g	=	Force at base of projectile due to gas pressure, i.e., $F_g = P_g \, A_b \, t^w$ (See Figure 4)	(lb)
I	=	Moment of inertia of projectile along its longitudinal axis	($lb \; sec^2 \; in$)
K_1	=	Constant of proportionality between T and $\delta \, A_{de}$, i.e., $T = K_1 \, \delta \, A_{de}$	(psi)
K_2	=	$\dfrac{K_1 \, N \tan \alpha}{2}$	(psi)
M	=	Mass of projectile	($lb \; sec^2/in$)
N	=	Number of lands or grooves of gun	
OP	=	Curve of projectile (See Figure 3)	
OR	=	Curve of rifling (See Figure 3)	
P_g	=	Gas pressure	(psi)

R	=	Driving edge force (See Figure 4)	(lb)
T	=	Torque that is developed as rotating band is forced to rotate through rifling due to firing	(in lb)
W	=	Wronskein = $-\dfrac{1}{\Gamma\left(1-\frac{1}{2(m+1)}\right)\Gamma\left(1+\frac{1}{2(m+1)}\right)}$	
b	=	Coefficient dependent on rifling twist, i.e., $y_r = bx^\beta$ (for uniform twist rifling $b = \frac{\pi}{n}$ and $\beta = 1$) (See Figure 3)	
k	=	$m - 2$	
m	=	Exponent dependent on relationship between x and t, i.e., $x = C_1 t^m$	
n	=	Constant of uniform twist rifling indicating number of bore diameters of longitudinal distance in which rifling of gun makes one complete revolution	
r	=	Radius of rifling $\frac{D_\ell}{2} < r < \frac{D_g}{2}$	
r_m	=	Mean of land and groove radii of gun	(in)
t	=	Time	(sec)
w	=	Exponent dependent on relationship between Pg and t, i.e., $P_g = C_2 t^w$	
x	=	Distance along longitudinal axis starting from origin of rifling	(in)
		Also, forward travel of projectile (at t = o, it is assumed that the forward end of the band is at the origin of rifling)	(in)
y_p	=	Arc length swept by projectile, i.e., $y_p = r_m \phi_p$ (See Figure 2)	(in)
y_r	=	Arc length swept by rifling, i.e., $y_r = r_m \phi_r$ (See Figures 2 and 3)	(in)
α	=	Angle that forcing cone makes with longitudinal axis of gun (See Figures 1 and 4)	(rad)
β	=	Exponent dependent on rifling twist, i.e., $y_r = bx^\beta$ (For uniform twist rifling $\beta = 1$) (See Figure 3)	

δ	=	Driving edge displacement, = arc length by which the projectile fails to follow the rifling, i.e., $\delta = y_r - y_p$ (See Figure 2)	(in)
θ_p	=	Angle that unrolled curve of projectile makes with longitudinal axis of gun (See Figure 4)	(rad)
θ_r	=	Angle that unrolled curve of rifling makes with longitudinal axis of gun (See Figures 3 and 4)	(rad)
μ	=	Coefficient of friction between band and gun	
$(\sigma_b)_{ave}$	=	Average driving edge pressure of a rotating band in the initial stages of engraving	(psi)
ϕ_p	=	Angle of rotation of projectile (See Figure 2)	(rad)
ϕ_r	=	Angle of rotation of rifling (See Figures 2 and 3)	(rad)

I. INTRODUCTION

In the past, driving edge pressures usually referred to those that occurred after full engravement of the band. However, in the case of a 105mm gun-projectile system (T140 E2 gun tube and TP-T79 E2 projectile), attention was focused to that region of the gun tube very close to the origin of rifling. In this region there was evidence of gun failure in the form of accelerated erosion rate at the driving edge of the lands of the rifling. Ensuing investigations disclosed that cracking and peeling of the chromium plating just beyond the origin of rifling on the driving edge side of the gun lands occurred after one round of firing. In fact, in a 105mm gun tube of the type referred to above, after only 73 rounds of firing, the rifling was so badly damaged that the gun lands had completely eroded away.

It was believed that high driving edge pressures in this region of initial engraving was a possible cause for the removal of the chromium deposit which, in turn, accelerated damage due to erosion. This report presents an analysis of this phenomenon and provides a general expression for determining driving edge pressures.

Unfortunately, some of the terms in the general equation are not precisely known. However, approximate values of these terms and, therefore, of driving edge pressures may be obtained by: a) assigning some reasonable value to the coefficient of friction between rotating band and gun tube; b) determining instantaneous values of both gas pressure and acceleration of the projectile from proper ballistic data or drawings; c) replacing the torque term of the general expression as a function of driving edge displacement instead of as a function of rotational acceleration of the projectile; and d) determining values of new terms introduced by this replacement.

Because uniform twist rifling is common, the above approach is carried out to completion for this type of rifling. For general type twist rifling, on the other hand, only the basic equations are presented. For clarity, the derivation work involved in the analysis has been presented in the Appendixes.

II. DISCUSSION AND RESULTS

It should be emphasized that the equations presented in this report yield only average driving edge pressure of a rotating band since the distribution of this pressure over the entire band is not known. However, this average pressure is still of extreme importance because it may be high enough to require a band of different design and/or a rifling coating of greater strength than may be called for by present design procedures.

Equation A5* is the most general expression for determining driving edge pressures, but it is in a form from which numerical values can be obtained

*The numbered equations referred to in this section are those that appear in Appendix A.

only with extreme difficulty. Part of the aim of this report has been to modify and reduce this equation so that solution of the resulting expressions may be more readily performed.

Equation A13 represents a modification of this general expression and has been derived by assuming that torque varies linearly with the product of driving edge displacement and the projected driving edge area of the band, i.e., $T = K_1 \delta (A_{de})_{proj}$.

This assumption results in the relationship that the torque, which is developed as the projectile first engages the rifling, is proportional to the product of the corresponding driving edge displacement and square of the axial distance travelled by the projectile, i.e., $T = K_2 \delta x^2$. In order to compute driving edge pressures by equations that are based upon the above relationship, the values of both K_2 and δ must first be determined. The constant of proportionality, K_2, will be discussed next, but discussion of driving edge displacement, δ, will be treated later in this section.

Unfortunately, no suitable analytical method is known for computing the value of K_2 which may be looked upon as a spring constant of a simple spring-mass system where the rotating band represents the spring, the projectile represents the mass, and the torque represents the forcing function on the spring. However, a reasonable value of this constant can be determined experimentally. In fact, by means of static push tests, a value of K_2 has been found for a 105mm gun-projectile system of the type mentioned in the Introduction of this report, and the details of this experimental work are available at the Theoretical and Applied Mechanics Branch of Watertown Arsenal Laboratories.

It is recognized that static push tests do not include the dynamic responses of the system and of the materials. Until suitable dynamic test results are available, however, static push test results must be relied upon.

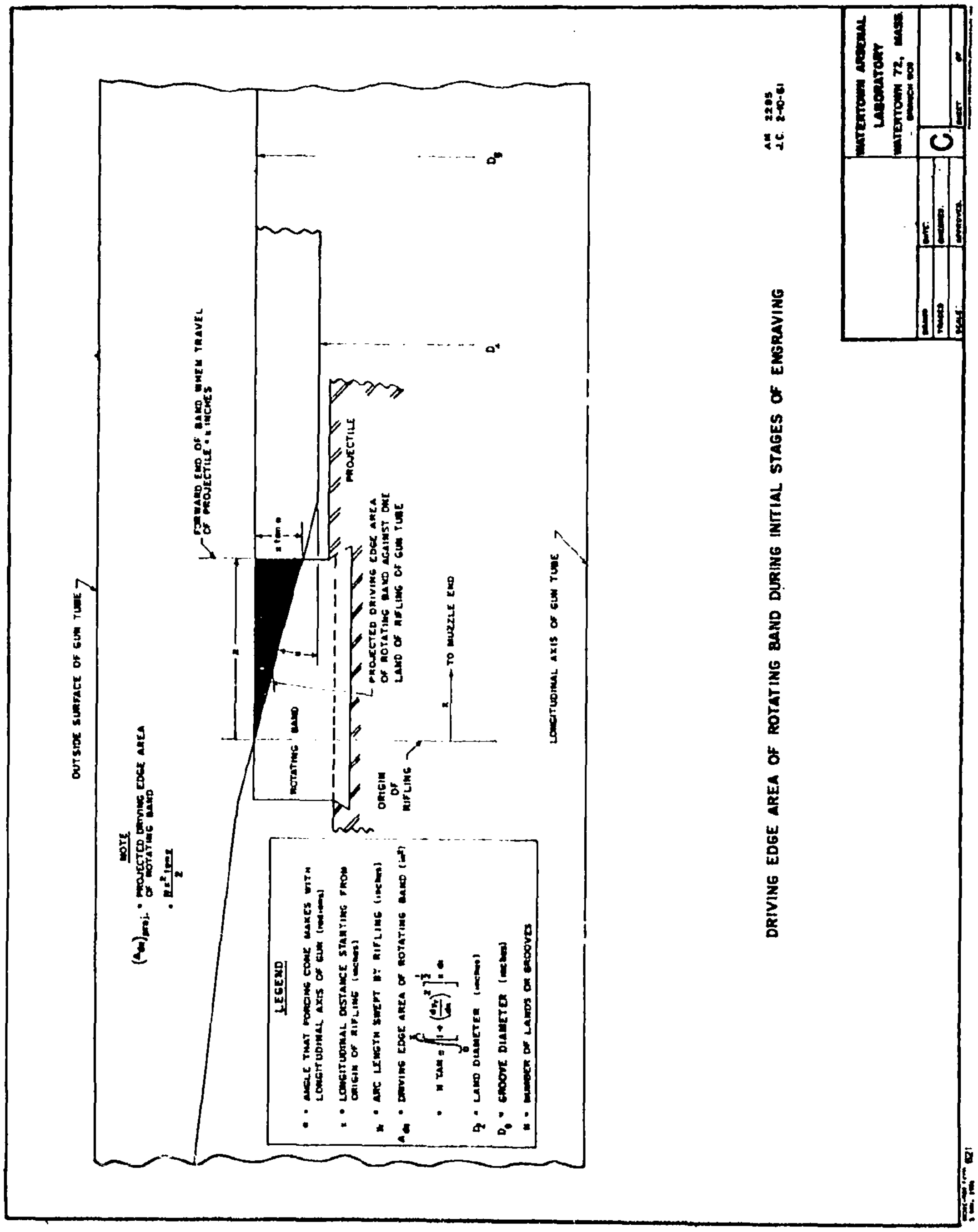
Equations A29 and A30 are modifications of Equations A5 and A13, respectively, restricted specifically to uniform twist rifling. All terms appearing in these equations have been presented such that their numerical values as well as those of driving edge pressures may be computed.

In this report no attempt has been made to establish an equation for determining the value of driving edge displacement, δ, for general type twist rifling. However, Equation A51, an expression for finding the value of δ (in terms of t), has been derived for the specific case of uniform twist rifling.

It is also assumed that when the front of the unengraved band is at the origin of rifling the projectile has zero acceleration, i.e., no free run.

It should be remembered that the equations presented in this report are valid, within the assumptions made, only for initial stages of engraving. For the case where the band is fully engraved, use of References 1 and 2 is recommended.

Finally, it must be recognized that further study is needed for more precise determination of initial driving edge pressures and that the equations presented in this report should be relied upon to give orders of magnitude rather than actual values.



DRIVING EDGE AREA OF ROTATING BAND DURING INITIAL STAGES OF ENGRAVING
OUTSIDE SURFACE OF GUN TUBE
FORWARD END OF BAND WHEN TRAVEL OF PROJECTILE = x INCHES
PROJECTILE
ROTATING BAND
PROJECTED DRIVING EDGE AREA OF ROTATING BAND AGAINST ONE LAND OF RIFLING OF GUN TUBE
TO MUZZLE END
ORIGIN OF RIFLING
LONGITUDINAL AXIS OF GUN TUBE
NOTE
LEGEND
LAND DIAMETER
GROOVE DIAMETER
NUMBER OF LANDS OR GROOVES
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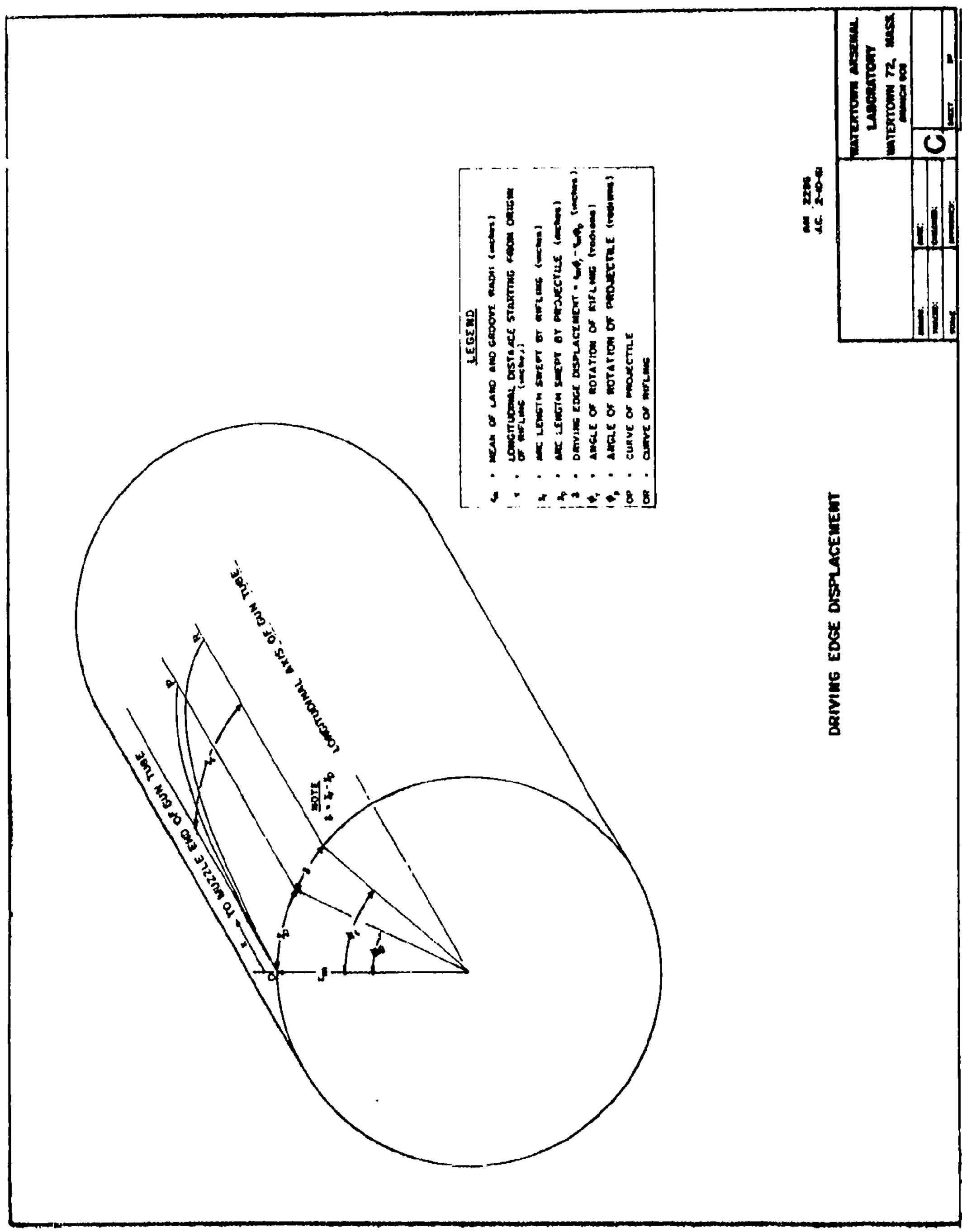
FIGURE 1



LEGEND
DRIVING EDGE DISPLACEMENT
LONGITUDINAL AXIS OF GUN TUBE
TO MUZZLE END OF GUN TUBE
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FIGURE 2

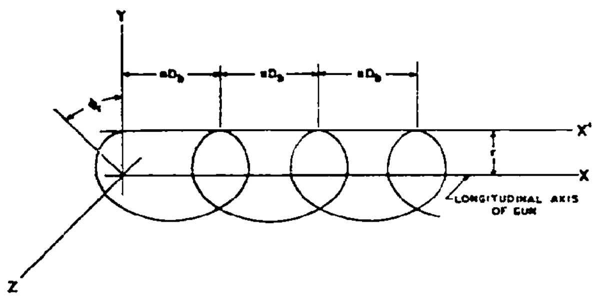

FIGURE 3A. SPACE CURVE OF RIFLING

LEGEND

- b = SLOPE OF UNROLLED RIFLING CURVE = $\tan\theta_r = \frac{\pi}{n}$
- n = NUMBER OF BORE DIAMETERS OF LONGITUDINAL DISTANCE IN WHICH RIFLING MAKES ONE COMPLETE REVOLUTION
- r = RADIUS OF RIFLING (inches)
- x = LONGITUDINAL DISTANCE STARTING FROM THE ORIGIN OF RIFLING (inches)
- y_r = ARC LENGTH SWEPT BY RIFLING (inches)
- D_b = BORE DIAMETER (inches)
- θ_r = ANGLE THAT UNROLLED CURVE OF RIFLING MAKES WITH LONGITUDINAL AXIS OF GUN (radians)
- ϕ_r = ANGLE OF ROTATION OF RIFLING (radians)

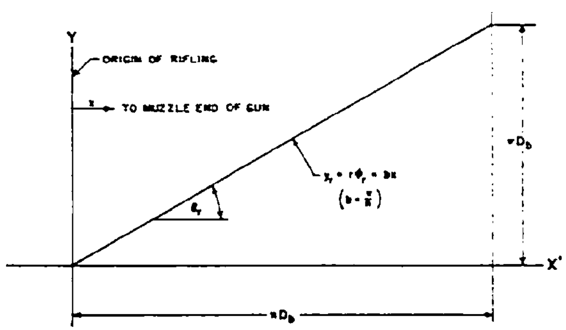

FIGURE 3B. UNROLLED RIFLING CURVE

UNIFORM TWIST RIFLING CURVES

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FIGURE 3

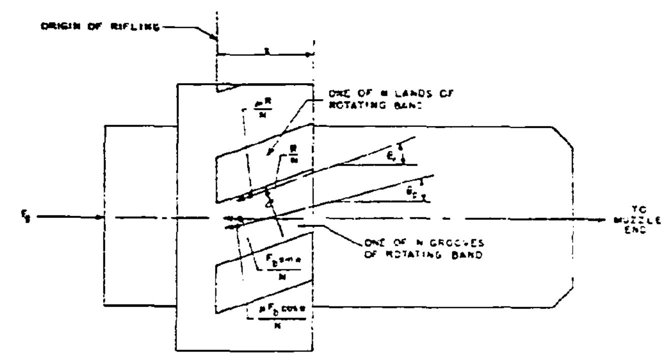
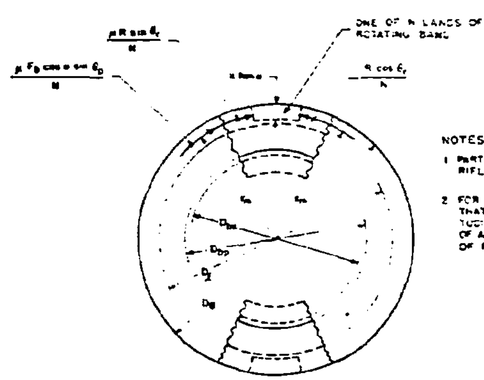


FIGURE 4A. FREE BODY DIAGRAM SHOWING FORCES AFFECTING FORWARD MOTION OF PROJECTILE



FREE BODY DIAGRAM SHOWING FORCES AFFECTING ROTATION OF PROJECTILE

LEGEND

- r_m = MEAN OF LAND AND GROOVE RADII (inches)
- x = LONGITUDINAL DISTANCE STARTING FROM THE ORIGIN OF RIFLING (inches)
- α = ANGLE THAT FORCING CONE MAKES WITH LONGITUDINAL AXIS OF GUN (radians)
- μ = COEFFICIENT OF FRICTION BETWEEN BAND AND GUN
- θ_r = ANGLE THAT UNROLLED RIFLING CURVE MAKES WITH LONGITUDINAL AXIS OF GUN (radians)
- θ_p = ANGLE THAT UNROLLED CURVE OF PROJECTILE MAKES WITH LONGITUDINAL AXIS OF GUN (radians)
- D_{bs} = DIAMETER OF BAND SEAT (inches)
- D_g = DIAMETER OF GROOVE OF GUN (inches)
- D_l = DIAMETER OF LAND OF GUN (inches)
- D_{bp} = DIAMETER OF BASE OF PROJECTILE (inches)
- F_B = FORCE ON OUTSIDE SURFACE OF BAND DUE TO BAND PRESSURE (pounds)
- F_G = FORCE AT BASE OF PROJECTILE DUE TO GAS PRESSURE (pounds)
- N = NUMBER OF LANDS OR GROOVES
- R = DRIVING EDGE FORCE (pounds)

NOTES

1. PARTIAL SECTION VIEW AT ORIGIN OF RIFLING LOOKING AT MUZZLE END
2. FOR EASE OF COMPUTATION IT IS ASSUMED THAT THE DISTANCE BETWEEN THE LONGITUDINAL AXIS OF PROJECTILE AND THE LINE OF ACTION OF FORCES AFFECTING ROTATION OF PROJECTILE IS ALWAYS EQUAL TO r_m

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FREE BODY DIAGRAMS OF PROJECTILE WITH PARTIALLY ENGRAVED ROTATING BAND

FIGURE 4

APPENDIX A

DERIVATION OF EQUATIONS FOR DETERMINING INITIAL DRIVING EDGE PRESSURES OF A ROTATING BAND

The general equations of motion are derived by considering all forces acting on the projectile, and simultaneous solution of these equations leads to a general expression for driving edge pressures. Simplifying assumptions are made to obtain expressions in presently usable form for the particular case of uniform twist rifling. For convenience, this section is divided into two parts: the first, applicable to general type twist rifling; the second, restricted specifically to uniform twist rifling.

PART I - GENERAL EQUATIONS FOR GENERAL TWIST RIFLING

General equations of motion

From the free body diagrams of Figure 4 the general equations of motion both in the axial and rotational directions may be quickly derived:

$$M \frac{d^2x}{dt^2} = F_g - R\ (\sin \theta_r + \mu \cos \theta_r) - F_b\ (\sin \alpha + \mu \cos \alpha \cos \theta_p) \tag{A1}$$

$$I \frac{d^2\phi_p}{dt^2} = r_m\ R\ (\cos \theta_r - \mu \sin \theta_r) - \mu\ r_m\ F_b \cos \alpha \sin \theta_p \tag{A2}$$

Driving edge force

Simultaneous solution of the above equations for driving edge force, R, leads to:

$$R = \frac{\left[F_g - M \frac{d^2x}{dt^2}\right]\left[\mu \sin \theta_p\right] + \left[\frac{I}{r_m}\ \frac{d\phi_p}{dt^2}\right]\left[\tan \alpha + \mu \cos \theta_p\right]}{\left[\cos \theta_r - \mu \sin \theta_r\right]\ \left[\tan \alpha + \mu \cos \theta_p\right] + \left[\sin \theta_r + \mu \cos \theta_r\right]\ \left[\mu \sin \theta_p\right]} \tag{A3}$$

Driving edge area

It can be shown (refer to Figure 1) that the total driving edge area of a rotating band, A_{de}, may be represented as:

$$A_{de} = N \tan \alpha \int_0^x \left[1 + \left(\frac{dy_r}{dx}\right)^2\right]^{\frac{1}{2}} x\ dx \tag{A4}$$

General expression for driving edge pressure

A general expression for determining average driving edge pressure may be obtained by dividing Equation A3 by Equation A4:

$$(\sigma_b)_{ave} = \frac{\left[F_g - M\frac{d^2x}{dt^2}\right]\left[\mu \sin\theta_p\right] + \left[\frac{I}{r_m}\;\frac{d^2\phi_p}{dt^2}\right]\left[\tan\alpha + \mu\cos\theta_p\right]}{N\tan\alpha\int_0^x\left[1+\left(\frac{dy_r}{dx}\right)^2\right]^{\frac{1}{2}} x\,dx\left\{\left[\cos\theta_r - \mu\sin\theta_r\right]\left[\tan\alpha + \mu\cos\theta_p\right] + \left[\sin\theta_r + \mu\cos\theta_r\right]\left[\mu\sin\theta_p\right]\right\}} \qquad \text{(A5)}$$

Torque expressed as a function of driving edge displacement

Unfortunately, the existence of the $\frac{d^2\phi_p}{dt^2}$ term in the numerator makes the above equation non-usable. However, torque may be defined as:

$$T = I\,\frac{d^2\phi_p}{dt^2} \qquad \text{(A6)}$$

Also, it is reasonable to assume that torque varies linearly with the product of driving edge displacement and projected driving edge area (see Figure 1), or that:

$$T = K_1\;\delta\;(A_{de})_{proj} \qquad \text{(A7)}$$

From Figure 1 it is seen that the projection of driving edge area may be expressed as:

$$(A_{de})_{proj} = \frac{N\,x^2\,\tan\alpha}{2} \qquad \text{(A8)}$$

If the above value for projected driving edge area is substituted for that shown in Equation A7, the following result will be obtained:

$$T = \frac{K_1\;\delta\;N\,x^2\,\tan\alpha}{2} \qquad \text{(A9)}$$

Modification of Equation A5

Since Equation A6 and A9 are both expressions of torque, the right hand sides of these equations may be set equal to each other to yield:

$$I\,\frac{d^2\phi_p}{dt^2} = \frac{K_1\;\delta\;N\,x^2\,\tan\alpha}{2} \qquad \text{(A10)}$$

The above equation may be rewritten as:

$$I \frac{d^2\phi_p}{dt^2} = K_2 \delta \, x^2 \qquad \text{(A11)}$$

where

$$K_2 = \frac{K_1 \, N \tan \alpha}{2} \qquad \text{(A12)}$$

The value for $I \dfrac{d^2\phi_p}{dt^2}$ given in Equation A11 may be substituted for that shown in Equation A5 to obtain a modification of the general expression:

$$(\sigma_b)_{ave} = \frac{\left[F_g - M\frac{d^2x}{dt^2}\right]\left[\mu \sin \theta_p\right] + \left[\frac{K_2}{r_m} \delta \, x^2\right]\left[\tan \alpha + \mu \cos \theta_p\right]}{N \tan \alpha \int_0^x \left[1 + \left(\frac{dy_r}{dx}\right)^2\right]^{\frac{1}{2}} x \, dx \left\{\left[\cos \theta_r - \mu \sin \theta_r\right]\left[\tan \alpha + \mu \cos \theta_p\right] + \left[\sin \theta_r + \mu \cos \theta_r\right]\left[\mu \sin \theta_p\right]\right\}} \qquad \text{(A13)}$$

PART II - GENERAL EQUATIONS FOR UNIFORM TWIST RIFLING

Since uniform twist rifling is most commonly used, modifications for this particular type rifling are made. Before these modifications can be presented, however, it becomes necessary to interrelate the following terms:

Terms containing x

x may be expressed as a function of t:

$$x = C_1 \, t^m \qquad \text{(A14)}$$

where the values of C_1 and m must be determined from the data of firing tests or from appropriate ballistic drawings.

Squaring of both sides of Equation A14 leads to:

$$x^2 = C_1^2 \, t^{2m} \qquad \text{(A15)}$$

First and second derivatives of Equation A14 with respect to t yield:

$$\frac{dx}{dt} = C_1 \, m \, t^{m-1} \tag{A16}$$

$$\frac{d^2x}{dt^2} = C_1 \, m \, (m - 1) \, t^{m-2} \tag{A17}$$

First derivative of t with respect to x, on the other hand, may be obtained by the inverse of Equation A16:

$$\frac{dt}{dx} = \frac{1}{C_1 \, m \, t^{m-1}} \tag{A18}$$

Also, from Equation A14, t may be expressed as a function of x:

$$t = \left(\frac{x}{C_1}\right)^{\frac{1}{m}} \tag{A19}$$

From Figure 3B it is seen that:

$$y_r = b \, x \tag{A20}$$

where

$$b = \frac{\pi}{n} \tag{A21}$$

<u>Terms containing θ_r</u>

Also from Figure 3B it is seen that:

$$\tan \theta_r = \frac{dy_r}{dx} = b \tag{A22}$$

$$\sin \theta_r = \frac{b}{(1 + b^2)^{\frac{1}{2}}} \tag{A23}$$

$$\cos \theta_r = \frac{1}{(1 + b^2)^{\frac{1}{2}}} \tag{A24}$$

The pertinent equations previously shown, now restricted to uniform twist rifling, may be rewritten:

General equations of motion

$$M \frac{d^2x}{dt^2} = F_g - \frac{R\ (b + \mu)}{(1 + b^2)^{\frac{1}{2}}} - F_b\ (\sin \alpha + \mu \cos \alpha \cos \theta_p) \tag{A25}$$

$$I \frac{d^2\phi_p}{dt^2} = r_m \left[\frac{R\ (1 - \mu\ b)}{(1 + b^2)^{\frac{1}{2}}} - \mu\ F_b \cos \alpha \sin \theta_p\right] \tag{A26}$$

Driving edge force

$$R = \frac{\left[1 + b^2\right]^{\frac{1}{2}} \left\{\left[F_g - M \frac{d^2x}{dt^2}\right] \left[\mu \sin \theta_p\right] + \left[\frac{I}{r_m} \frac{d^2\phi_p}{dt^2}\right] \left[\tan \alpha + \mu \cos \theta_p\right]\right\}}{\left[1 - \mu\ b\right] \left[\tan \alpha + \mu \cos \theta_p\right] + \left[b + \mu\right] \left[\mu \sin \theta_p\right]} \tag{A27}$$

Driving edge area

$$A_{de} = \frac{Nx^2}{2} \tan \alpha\ (1 + b^2)^{\frac{1}{2}} \tag{A28}$$

Modification of Equation A5

$$(\sigma_b)_{ave} = \frac{2\left\{\left[F_g - M \frac{d^2x}{dt^2}\right] \left[\mu \sin \theta_p\right] + \left[\frac{I}{r_m} \frac{d^2\phi_p}{dt^2}\right] \left[\tan \alpha + \mu \cos \theta_p\right]\right\}}{Nx^2 \tan \alpha \left\{\left[1 - \mu b\right] \left[\tan \alpha + \mu \cos \theta_p\right] + \left[b + \mu\right] \left[\mu \sin \theta_p\right]\right\}} \tag{A29}$$

Modification of Equation A13

$$(\sigma_b)_{ave} = \frac{2\left\{\left[F_g - M \frac{d^2x}{dt^2}\right] \left[\mu \sin \theta_p\right] + \left[\frac{K_2}{r_m} \delta\ x^2\right] \left[\tan \alpha + \mu \cos \theta_p\right]\right\}}{Nx^2 \tan \alpha \left\{\left[1 - \mu b\right] \left[\tan \alpha + \mu \cos \theta_p\right] + \left[b + \mu\right] \left[\mu \sin \theta_p\right]\right\}} \tag{A30}$$

The remainder of this section will supply additional interrelations of terms that appear in the derived equations, applicable to uniform twist rifling, so that the values of these individual terms as well as those of driving edge pressures may be computed.

Gas pressure force

P_g may be expressed as a function of t:

$$P_g = C_2 \, t^w \tag{A31}$$

where, once more, the values of C_2 and w must also be determined from the data of firing tests or from appropriate ballistic drawings.

Then,

$$F_g = A_b \, P_g = C_2 \, A_b \, t^w \tag{A32}$$

Value of θ_p

By definition:

$$y_p = y_r - \delta \tag{A33}$$

The tangent of θ_p may be expressed as:

$$\tan \theta_p = \frac{dy_p}{dx} = \frac{dy_r}{dx} - \frac{d\delta}{dx} \tag{A34}$$

The last term of the above equation may also be represented as:

$$\frac{d\delta}{dx} = \frac{d\delta}{dt} \cdot \frac{dt}{dx} \tag{A35}$$

If the value of $\frac{dt}{dx}$ shown in Equation A18 is substituted into the above equation, it becomes:

$$\frac{d\delta}{dx} = \frac{\frac{d\delta}{dt}}{C_1 \, m \, t^{m-1}} \tag{A36}$$

Substitution of the above value of $\frac{d\delta}{dx}$ as well as the value of $\frac{dy_r}{dx}$ shown in Equation A22 into Equation A34 yields:

$$\tan \theta_p = b - \frac{\frac{d\delta}{dt}}{C_1 \, m \, t^{m-1}} \tag{A37}$$

Therefore,

$$\theta_p = \tan^{-1} \left[b - \frac{\frac{d\delta}{dt}}{C_1 \, m \, t^{m-1}} \right] \tag{A38}$$

Value of δ

ϕ_p may be expressed as:

$$\phi_p = \frac{1}{r_m} y_p = \frac{1}{r_m} (y_r - \delta) \tag{A39}$$

First and second derivatives of the above equation lead to:

$$\frac{d\phi_p}{dt} = \frac{1}{r_m}\left(\frac{dy_r}{dt} - \frac{d\delta}{dt}\right) \tag{A40}$$

$$\frac{d^2\phi_p}{dt^2} = \frac{1}{r_m}\left(\frac{d^2y_r}{dt^2} - \frac{d^2\delta}{dt^2}\right) \tag{A41}$$

Multiplying both sides of Equation A11 by $\frac{r_m}{I}$ yields:

$$r_m \frac{d^2\phi_p}{dt^2} = \frac{K_2 \ \delta \ r_m \ x^2}{I} \tag{A42}$$

If the value of $\frac{d^2\phi_p}{dt^2}$ shown in Equation A41 and the value of x^2 shown in equation A15 are substituted into the above equation, it becomes:

$$\left(\frac{d^2y_r}{dt^2} - \frac{d^2\delta}{dt^2}\right) = \frac{K_2 \ \delta \ r_m \ C_1^2 \ t^{2m}}{I} \tag{A43}$$

First and second derivatives of Equation A20 yield:

$$\frac{dy_r}{dt} = b \frac{dx}{dt} \tag{A44}$$

$$\frac{d^2y_r}{dt^2} = b \frac{d^2x}{dt^2} \tag{A45}$$

If the value of $\frac{d^2x}{dt^2}$ shown in Equation A17 is substituted into the above equation, it becomes:

$$\frac{d^2y_r}{dt^2} = b \ C_1 \ m \ (m - 1) \ t^{m-2} \tag{A46}$$

If the above value of $\frac{d^2y_r}{dt^2}$ is substituted into Equation A43, it may be rewritten as:

$$\frac{d^2\delta}{dt^2} + \frac{K_2\delta r_m C_1^2 t^{2m}}{I} = bC_1 m(m-1)t^{m-2} \tag{A47}$$

Let

$$A = \frac{K_2 r_m C_1^2}{I} \tag{A48}$$

and

$$B = bC_1 m(m-1) \tag{A49}$$

Then Equation A47 becomes:

$$\frac{d^2\delta}{dt^2} + A\delta t^{2m} = Bt^{m-2} \tag{A50}$$

Solution of the above equation for δ, the details of which are given in Appendix B, leads to:

$$\delta = \frac{B}{W}\left\{\left[\sum_{r=o}^{\infty} \frac{(-1)^r\left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r-\frac{1}{2(m+1)}}}{r!\left(r-\frac{1}{2(m+1)}\right)!}\, t^{2r(m+1)} \cdot \sum_{r=o}^{\infty} \frac{(-1)^r\left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r+\frac{1}{2(m+1)}}}{r!\left(r+\frac{1}{2(m+1)}\right)!}\, \frac{t^{2r(m+1)+m}}{2r(m+1)+m}\right] - \left[\sum_{r=o}^{\infty} \frac{(-1)^r\left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r+\frac{1}{2(m+1)}}}{r!\left(r+\frac{1}{2(m+1)}\right)!}\, t^{2r(m+1)+1} \right.\right. \tag{A51}$$

$$\left.\left.\cdot \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r - \frac{1}{2(m+1)}}}{r!\left(r - \frac{1}{2(m+1)}\right)!} \; \frac{t^{2r(m+1)+m-1}}{2r(m+1)+m-1}\right]\right\} \tag{A51}$$

where

$$W = -\frac{1}{\Gamma\left(1 - \frac{1}{2(m+1)}\right)\Gamma\left(1 + \frac{1}{2(m+1)}\right)} \tag{A52}$$

Value of $\frac{d^2\delta}{dt^2}$

First derivative of Equation A51 with respect to t yields:

$$\begin{aligned}
\frac{d\delta}{dt} = \frac{B}{W} \Bigg\{ \Bigg[& \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r - \frac{1}{2(m+1)}}}{r!\left(r - \frac{1}{2(m+1)}\right)!} \, [2r(m+1)]\, t^{2r(m+1)-1} \\
& \cdot \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r + \frac{1}{2(m+1)}}}{r!\left(r + \frac{1}{2(m+1)}\right)!} \, t^{2r(m+1)+m-1} \Bigg] \\
& - \Bigg[\sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r + \frac{1}{2(m+1)}}}{r!\left(r + \frac{1}{2(m+1)}\right)!} \, [2r(m+1)+1]\, t^{2r(m+1)} \\
& \cdot \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2(m+1)}\right)^{2r - \frac{1}{2(m+1)}}}{r!\left(r - \frac{1}{2(m+1)}\right)!} \, t^{2r(m+1)+m} \Bigg] \Bigg\}
\end{aligned} \tag{A53}$$

References 3 and 4 were relied upon in order to obtain the solutions presented in this section.

APPENDIX B

SOLUTION OF DRIVING EDGE DISPLACEMENT AS A FUNCTION OF TIME FOR UNIFORM TWIST RIFLING

In order not to detract from the continuity of the derivation presented in Appendix A of this report, the detailed steps involved in the solution of Equation A50 were omitted. These steps, however, which have been made self-explanatory, are presented in this section.

Equation to be solved

$$\frac{d^2\delta}{dt^2} + A\delta t^{2m} = Bt^{m-2} \tag{B1}$$

where

$$A = \frac{K_2 r_m C_1^2}{I} \tag{B2}$$

and

$$B = bC_1 m(m-1) \tag{B3}$$

Let

$$m - 2 = k \tag{B4}$$

$$\frac{d\delta}{dt} = \delta' \tag{B5}$$

and

$$\frac{d^2\delta}{dt^2} = \delta'' \tag{B6}$$

Initial conditions

$$\text{at } t = o, \text{ both } \delta = o \text{ and } \delta' = o \tag{B7}$$

Homogeneous solution

$$\delta'' + At^{2m}\delta = o \tag{B8}$$

$$\delta = t^{\frac{1}{2}} J_{+\frac{1}{2m+2}}\left(\frac{\sqrt{A}}{m+1}\, t^{m+1}\right) \tag{B9}$$

$$\delta = c_1 t^{\frac{1}{2}} J_{\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1} \right) + c_2 t^{\frac{1}{2}} J_{-\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1} \right) \qquad \text{(B10)}$$

Variation of parameters

$$\delta'' + At^{2m}\delta = Bt^k \qquad \text{(B11)}$$

$$\delta = c_1(t)\delta_1(t) + c_2(t)\delta_2(t) \qquad \text{(B12)}$$

where

$$\delta_1(t) = t^{\frac{1}{2}} J_{\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1} \right) \qquad \text{(B13)}$$

and

$$\delta_2(t) = t^{\frac{1}{2}} J_{-\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1} \right) \qquad \text{(B14)}$$

Then

$$\delta' = c_1'\delta_1 + c_2'\delta_2 + c_1\delta_1' + c_2\delta_2', \qquad \text{(B15)}$$

$$\delta'' = c_1'\delta_1' + c_2'\delta_2' + c_1\delta_1'' + c_2\delta_2'', \qquad \text{(B16)}$$

and

$$c_1'\delta_1' + c_2'\delta_2' + c_1\delta_1'' + c_2\delta_2'' + c_1At^{2m}\delta_1 + c_2At^{2m}\delta_2 = Bt^k \qquad \text{(B17)}$$

for it is assumed that:

$$c_1'\delta_1 + c_2'\delta_2 = o \qquad \text{(B18)}$$

Now

$$c_1'\delta_1' + c_2'\delta_2' = Bt^k \qquad \text{(B19)}$$

Then

$$c_1' = \frac{\begin{vmatrix} 0 & \delta_2 \\ Bt^k & \delta_2' \end{vmatrix}}{\begin{vmatrix} \delta_1 & \delta_2 \\ \delta_1' & \delta_2' \end{vmatrix}} = -\frac{\delta_2 Bt^k}{W} \tag{B20}$$

$$c_2' = \frac{\begin{vmatrix} \delta_1 & 0 \\ \delta_1' & Bt^k \end{vmatrix}}{W} = \frac{\delta_1 Bt^k}{W}, \tag{B21}$$

$$c_1 = -\frac{B}{W} \int_0^t \delta_2 t^k dt \tag{B22}$$

and

$$c_2 = \frac{B}{W} \int_0^t \delta_1 t^k dt \tag{B23}$$

(It can be shown that in this case the value of the Wronskein, W, is a constant.)

$$\delta = \delta_1(t) \left[-\frac{B}{W}\right] \int_0^t \delta_2(s) s^k ds + \delta_2(t) \left[\frac{B}{W}\right] \int_0^t \delta_1(s) s^k ds \tag{B24}$$

$$\delta = \frac{B}{W} \int_0^t s^k [\delta_1(s)\delta_2(t) - \delta_2(s)\delta_1(t)] ds \tag{B25}$$

$$\begin{aligned} \delta = \frac{B}{W} \int_0^t s^k \Bigg[& s^{\frac{1}{2}} J_{\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1}\right) t^{\frac{1}{2}} J_{-\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1}\right) \\ & - s^{\frac{1}{2}} J_{-\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1}\right) t^{\frac{1}{2}} J_{\frac{1}{2m+2}} \left(\frac{\sqrt{A}}{m+1} t^{m+1}\right) \Bigg] ds \end{aligned} \tag{B26}$$

$$\frac{\sqrt{A}}{m+1} s^{m+1} = Q(s) \tag{B27}$$

$$\frac{\sqrt{A}}{m+1} t^{m+1} = Q(t) \tag{B28}$$

$$\begin{aligned} \delta = \frac{B}{W} \int_{s=0}^{t} s^{k+\frac{1}{2}} t^{\frac{1}{2}} \Big\{ & J_{\frac{1}{2m+2}} [Q(s)] \; J_{-\frac{1}{2m+2}} [Q(t)] \\ & - J_{-\frac{1}{2m+2}} [Q(s)] \; J_{\frac{1}{2m+2}} [Q(t)] \Big\} ds \end{aligned} \tag{B29}$$

$$W = \delta_1 \delta_2' - \delta_2 \delta_1' \tag{B30}$$

Let

$$\frac{\sqrt{A}}{m+1} = \alpha \tag{B31}$$

$$\delta_1 = t^{\frac{1}{2}} J_{\frac{1}{2m+2}} \left(\alpha t^{m+1}\right), \quad \delta_2 = t^{\frac{1}{2}} J_{-\frac{1}{2m+2}} \left(\alpha t^{m+1}\right) \tag{B32}$$

$$J_{\frac{1}{2m+2}} (z) \sim \frac{\left(\frac{1}{2}z\right)^{\frac{1}{2m+2}}}{\Gamma\left(\frac{1}{2m+2} + 1\right)}, \quad J_{-\frac{1}{2m+2}} (z) \sim \frac{\left(\frac{1}{2}z\right)^{-\frac{1}{2m+2}}}{\Gamma\left(1 - \frac{1}{2m+2}\right)} \tag{B33}$$

$$J'_{\frac{1}{2m+2}}(z) \sim \frac{\frac{1}{2m+2}\left(\tfrac{1}{2}z\right)^{\frac{1}{2m+2}-1}}{\Gamma\left(\frac{1}{2m+2}+1\right)},$$

$$J_{-\frac{1}{2m+2}}(z) \sim \frac{-\frac{1}{2m+2}\left(\tfrac{1}{2}z\right)^{-\frac{1}{2m+2}-1}}{\Gamma\left(1-\frac{1}{2m+2}\right)} \tag{B34}$$

$$\delta_1 = t^{\frac{1}{2}}\, J_{\frac{1}{2m+2}}\left(\alpha t^{m+1}\right) \sim \frac{t^{\frac{1}{2}}\left(\tfrac{1}{2}\alpha t^{m+1}\right)^{\frac{1}{2m+2}}}{\Gamma\left(\frac{1}{2m+2}+1\right)} \sim \frac{\left(\tfrac{1}{2}\alpha\right)^{\frac{1}{2m+2}}}{\Gamma\left(\frac{1}{2m+2}+1\right)}\, t \tag{B35}$$

$$\delta_2 = t^{\frac{1}{2}}\, J_{-\frac{1}{2m+2}}\left(\alpha t^{m+1}\right) \sim \frac{t^{\frac{1}{2}}\left(\tfrac{1}{2}\alpha t^{m+1}\right)^{-\frac{1}{2m+2}}}{\Gamma\left(1-\frac{1}{2m+2}\right)} \sim \frac{\left(\tfrac{1}{2}\alpha\right)^{-\frac{1}{2m+2}}}{\Gamma\left(1-\frac{1}{2m+2}\right)} \tag{B36}$$

$$\delta_1' \simeq \frac{\left(\tfrac{1}{2}\alpha\right)^{\frac{1}{2m+2}}}{\Gamma\left(\frac{1}{2m+2}+1\right)}, \quad \delta_2' = 0 \tag{B37}$$

$$W = -\frac{\left(\tfrac{1}{2}\alpha\right)^{-\frac{1}{2m+2}}\left(\tfrac{1}{2}\alpha\right)^{\frac{1}{2m+2}}}{\Gamma\left(\frac{1}{2m+2}+1\right)\Gamma\left(1-\frac{1}{2m+2}\right)} = -\frac{1}{\Gamma\left(\frac{1}{2m+2}+1\right)\Gamma\left(1-\frac{1}{2m+2}\right)} \tag{B38}$$

$$\delta = \frac{B}{W}\left[\delta_2(t)\int_{s=0}^{t} s^{k+\frac{1}{2}} J_{\frac{1}{2m+2}}(\alpha s^m + 1)\, ds - \delta_1(t)\int_{s=0}^{t} s^{k+\frac{1}{2}} J_{-\frac{1}{2m+2}}(\alpha s^m + 1)\, ds\right] \quad \text{(B39)}$$

$$J_{\frac{1}{2m+2}}(z) = \sum_{r=0}^{\infty} \frac{\left(\frac{1}{2}z\right)^{2r+\frac{1}{2m+2}}(-1)^r}{r!\left(r+\frac{1}{2m+2}\right)!} \quad \text{(B40)}$$

$$J_{-\frac{1}{2m+2}}(z) = \sum_{r=0}^{\infty} \frac{\left(\frac{1}{2}z\right)^{2r-\frac{1}{2m+2}}(-1)^r}{r!\left(r-\frac{1}{2m+2}\right)!} \quad \text{(B41)}$$

$$s^{k+\frac{1}{2}} J_{\frac{1}{2m+2}}(\alpha s^m + 1) = \sum_{r=0}^{\infty} \frac{(-1)^r\left(\frac{\alpha s^m + 1}{2}\right)^{2r+\frac{1}{2m+2}}}{r!\left(r+\frac{1}{2m+2}\right)!} s^{k+\frac{1}{2}} \quad \text{(B42)}$$

$$s^{k+\frac{1}{2}} J_{\frac{1}{2m+2}}(\alpha s^m + 1) = \sum_{r=0}^{\infty} \frac{(-1)^r\left(\frac{\alpha}{2}\right)^{2r+\frac{1}{2m+2}}}{r!\left(r+\frac{1}{2m+2}\right)!} s^{2r(m+1)+k+1} \quad \text{(B43)}$$

$$\int_{s=0}^{t} s^{k+\frac{1}{2}} J_{\frac{1}{2m+2}}(\alpha s^m + 1)\, ds = \sum_{r=0}^{\infty} \frac{(-1)^r\left(\frac{\alpha}{2}\right)^{2r+\frac{1}{2m+2}}}{r!\left(r+\frac{1}{2m+2}\right)!} \int_{s=0}^{t} s^{2r(m+1)+k+1}\, ds \quad \text{(B44)}$$

$$\int_{s=0}^{t} s^{k+\frac{1}{2}} J_{\frac{1}{2m+2}} (\alpha s^{m+1}) ds = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r+\frac{1}{2m+2}}}{r! \left(r + \frac{1}{2m+2}\right)!} \frac{t^{2r(m+1)+k+2}}{2r(m+1)+k+2} \tag{B45}$$

$$s^{k+\frac{1}{2}} J_{-\frac{1}{2m+2}} (\alpha s^{m+1}) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha s^{m+1}}{2}\right)^{2r-\frac{1}{2m+2}}}{r! \left(r - \frac{1}{2m+2}\right)!} s^{k+\frac{1}{2}} \tag{B46}$$

$$s^{k+\frac{1}{2}} J_{-\frac{1}{2m+2}} (\alpha s^{m+1}) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r-\frac{1}{2m+2}}}{r! \left(r - \frac{1}{2m+2}\right)!} s^{2r(m+1)+k} \tag{B47}$$

$$\int_{s=0}^{t} s^{k+\frac{1}{2}} J_{-\frac{1}{2m+2}} (\alpha s^{m+1}) ds = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r-\frac{1}{2m+2}}}{r! \left(r - \frac{1}{2m+2}\right)!} \int_{s=0}^{t} s^{2r(m+1)+k} ds \tag{B48}$$

$$\int_{s=0}^{t} s^{k+\frac{1}{2}} J_{-\frac{1}{2m+2}} (\alpha s^{m+1}) ds = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r-\frac{1}{2m+2}}}{r! \left(r - \frac{1}{2m+2}\right)!} \frac{t^{2r(m+1)+k+1}}{2r(m+1)+k+1} \tag{B49}$$

$$\delta_1(t) = t^{\frac{1}{2}} J_{\frac{1}{2m+2}} (\alpha t^{m+1}) \tag{B50}$$

$$\delta_2(t) = t^{\frac{1}{2}} J_{-\frac{1}{2m+2}} (\alpha t^{m+1}) \tag{B51}$$

$$\delta_1(t) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha t^{m+1}}{2}\right)^{2r + \frac{1}{2m+2}}}{r! \left(r + \frac{1}{2m+2}\right)!} t^{\frac{1}{2}} \tag{B52}$$

$$\delta_1(t) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r + \frac{1}{2m+2}}}{r! \left(r + \frac{1}{2m+2}\right)!} t^{2r(m+1)+1} \tag{B53}$$

$$\delta_2(t) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha t^{m+1}}{2}\right)^{2r - \frac{1}{2m+2}}}{r! \left(r - \frac{1}{2m+2}\right)!} t^{\frac{1}{2}} \tag{B54}$$

$$\delta_2(t) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r - \frac{1}{2m+2}}}{r!\left(r - \frac{1}{2m-2}\right)!} t^{2r(m+1)} \qquad \text{(B55)}$$

$$\delta = \frac{B}{W}\left\{\left[\sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r - \frac{1}{2m+2}}}{r!\left(r - \frac{1}{2m+2}\right)!} t^{2r(m+1)} \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r + \frac{1}{2m+2}}}{r!\left(r + \frac{1}{2m+2}\right)!} \frac{t^{2r(m+1)+k+2}}{2r(m+1)+k+2}\right] - \left[\sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r + \frac{1}{2m+2}}}{r!\left(r + \frac{1}{2m+2}\right)!} t^{2r(m+1)+1} \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\alpha}{2}\right)^{2r - \frac{1}{2m+2}}}{r!\left(r - \frac{1}{2m+2}\right)!} \frac{t^{2r(m+1)+k+1}}{2r(m+1)+k+1}\right]\right\} \qquad \text{(B56)}$$

Finally:

$$\delta = \frac{B}{W}\left\{\left[\sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2m+2}\right)^{2r - \frac{1}{2m+2}}}{r!\left(r - \frac{1}{2m+2}\right)!} t^{2r(m+1)} \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2m+2}\right)^{2r + \frac{1}{2m+2}}}{r!\left(r + \frac{1}{2m+2}\right)!} \frac{t^{2r(m+1)+m}}{2r(m+1)+m}\right] - \left[\sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2m+2}\right)^{2r + \frac{1}{2m+2}}}{r!\left(r + \frac{1}{2m+2}\right)!} t^{2r(m+1)+1} \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{\sqrt{A}}{2m+2}\right)^{2r - \frac{1}{2m+2}}}{r!\left(r - \frac{1}{2m+2}\right)!} \frac{t^{2r(m+1)+m-1}}{2r(m+1)+m-1}\right]\right\} \qquad \text{(B57)}$$

The above solution is that which is shown as Equation A51 in Appendix A.

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Driving edge pressures of a rotating band during initial stages of engraving may be sufficiently high to produce cracking of chromium plating and subsequent accelerated gun tube erosion in the region of the origin of rifling. In this report the general differential equations of motion defining the force equilibrium in the axial and rotational directions as a function of time have been derived. Simultaneous solution of these equations leads to a general expression, applicable to general twist rifling, for determining initial driving edge pressures. Further modifications of this expression are made specifically for uniform twist rifling. Finally, additional interrelations of terms appearing in the equations are supplied so that these individual terms as well as those of driving edge pressures may be computed.

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Driving edge pressures of a rotating band during initial stages of engraving may be sufficiently high to produce cracking of chromium plating and subsequent accelerated gun tube erosion in the region of the origin of rifling. In this report the general differential equations of motion defining the force equilibrium in the axial and rotational directions as a function of time have been derived. Simultaneous solution of these equations leads to a general expression, applicable to general twist rifling, for determining initial driving edge pressures. Further modifications of this expression are made specifically for uniform twist rifling. Finally, additional interrelations of terms appearing in the equations are supplied so that these individual terms as well as those of driving edge pressures may be computed.

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